ENERGY PARTITIONING FOR SEISMIC EVENTS IN FENNOSCANDIA AND NW RUSSIA

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ABSTRACT

Close cooperation has been maintained with the operators of the Pyhäsalmi mine in central Finland, where an in-mine network of 16 sensors (4 three-component and 12 vertical-component geophones) has been operational since January 2003. Up to the end of April 2004, waveforms for more than 18,000 rockbursts and mining blasts have been collected.

Using data processing software, developed at NORSAR, we have automatically estimated the seismic moment and corner frequency for all events from the shear wave signal and its coda. The results show some scattering, but also three distinct clusters. One cluster is found at frequencies between 100 and 800 Hz with a seismic moment between 10^{10} and 10^{8} Nm (corresponding to moment magnitudes between $M_{\rm w}$ 0.6 and -0.73). Most of the events within this cluster could be identified as daily production ripple-fired blasts at around 10 p.m. local time. A second cluster has corner frequencies between 100 and 400 Hz and seismic moments between 10^{7} and 10^{9} ($M_{\rm w}$ 0 to -1.4). The associated events are mainly microseismic events and small rockbursts that occur at random times during the day. A third small cluster is found at corner frequencies between 20 and 40 Hz and having seismic moments similar to the production blasts. Most of these events occurred prior to, and in the same area as, the largest rockburst ($M_{\rm w}$ 1.1) that occurred on 26 January 2003. This suggests that these events may be associated with a gradual failure of the pillar involved.

In order to assess the source contribution to the observed shear wave energy, we have calculated S/P ratios for both production blasts and rockbursts. A data set of about 2,000 well-located events with recordings at a minimum of 3 three-component stations has been selected. As expected, the rockbursts show generally larger S/P ratios, and we are continuing to investigate the effect of signal frequency and source location on the estimated ratios. However, significant shear energy is clearly observed within 100 meters of the source for both microearthquakes (rockbursts) and mine blasts.

Based on detailed information on the geometry of the 1,500 m deep Pyhäsalmi mine, we have constructed a three-dimensional (3-D) gridded model that includes the velocity and density characteristics of the ore bodies, mined-out voids, access tunnels, and surrounding rocks. Using the E3D finite difference code developed at Lawrence Livermore National Laboratory, extensive simulations of the wave propagation have been made in the mine model.

The modeling experiments have shown that we are able to model observed data at the Pyhäsalmi mine with a reasonable level of fit. Our favorable comparisons between synthetic and observed data lend a certain degree of confidence to our modeling techniques and our 3-D geologic model of the mine, which has been sampled with grid spacings of both 4 and 2 meters. Through simulations of the response to explosive sources, we have shown that significant shear energy can be generated in a 10-20 meter region around the source. This mode-converted energy can be of a similar amplitude to the incident compressional phase. Both the geologic heterogeneity and the structural influences of the mine appear to be contributors to the near-source generation of shear energy.

OBJECTIVE

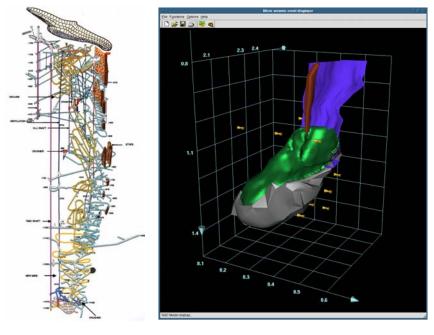
The main objective of this project is to increase nuclear explosion monitoring effectiveness through improved understanding of basic earthquake and explosion phenomenology. What this entails in essence is detailed characterization and understanding of how the seismic energy is generated from these phenomena (including simple and complex explosions, rockbursts, i.e. stress release in mines, and ordinary tectonic earthquakes, all at different depths and in different geological environments) and how this energy is partitioned between P and S waves. Specific questions are:

- How is the generation and partitioning of seismic energy affected by properties such as source region medium and overburden, the local structure, and the surrounding tectonic structure?
- What are the significant measurable effects of the partitioning of the seismic energy into various regional P and S phases, especially at higher frequencies?
- What is the physical basis for a measurable property, such as magnitude, that can be directly related to the yield of a fully coupled explosion, and how can emplacement conditions affect the observations?

RESEARCH ACCOMPLISHED

Introduction

This project is a three-year effort that started on 30 September 2002, and is a collaboration between NORSAR (as the lead organization) and Lawrence Livermore National Laboratory (LLNL). The work addresses the generation of seismic shear waves by explosions. During the last year we have focused on two main activities related to the Pyhäsalmi mine in central Finland (see Figure 1). The first activity has been concerned with analysis of data from the 16 sensor in-mine network. Up to the end of April 2004 waveforms for more than 18,000 rockbursts and mining blasts were collected, and both automatic and manual analyses has been conducted on this data set. The second main activity has been concerned with three-dimensional finite-difference calculations of the wavefield in a detailed model of the mine, using the E3D code developed at LLNL (Larsen & Schultz, 1995). Based on a the model obtained from the mine operator, the NORSAR 3-D Model Builder (Vinje et. al., 1999) has been used for building 3-D gridded velocity cubes that have been used by E3D. Velocity models with grid spacings of both 4 and 2 meters have been constructed.



part of the figure shows the shafts and access tunnels of the Pyhäsalmi mine from the surface down to a depth of 1,500 meters. The right-hand part of the figure shows a 3dimensional model of the mine, for the depth range 800-1,500 meters. Green represents copper ore, grey represents zinc ore, and purple and red represent backfilled material. The locations of the in-mine monitoring network are indicated by the vellow symbols.

Figure 1. The left-hand

Estimation of source parameters for events in the Pyhäsalmi mine

An automatic estimation of source parameters was conducted for the entire Pyhäsalmi data set that now comprises about 18,000 seismic events (rockbursts and blasts). For each event we analyzed the seismogram section containing the shear wave signal and its coda. Typically we chose a time window of 0.1 to 0.2 second duration which started at the automatically picked S-wave arrival. The geophones of the in-mine network have a natural frequency of 4.5 Hz, and we assume that the instrument response is flat to velocity for frequencies above 10 Hz. The seismogram segments were fourier-transformed to obtain the signal amplitude spectrum.

The observed signal spectra for a given seismic event depend on the source spectrum, the source radiation pattern, the receiver site effects, and propagation effects such as geometrical spreading and intrinsic attenuation. We multiplied the signal spectra by the source-receiver distance and compensated for the intrinsic attenuation. This procedure ideally provides the signal spectra at unit distance. We assumed a quality factor Q=800, which is a reasonable choice for the hard-rock conditions. In order to obtain a representative source spectrum, these spectra were averaged to eliminate the station effects and the source radiation pattern. One method of determining the basic source parameters, i.e. seismic moment and corner frequency, from the displacement spectra is to fit a Brune-type spectrum. Alternatively the corner frequency and seismic moment can be estimated by numerical integration of the displacement and velocity power spectrum (Andrews, 1986; Richardson & Jordan, 2002). For automatic processing of the entire data set, we applied the robust numerical integration method.

Figure 2 shows the seismic moment as a function of corner frequency for about 4,000 explosions and about 1,000 rockbursts. The event localization as well as the spectral analyses were done fully automatically, this time using the spectral fit method, but for this data set both the data and the processing results were quality-checked by visual inspection. The results show some scattering, but also three distinct clusters. One cluster (green symbols) is at frequencies between 100 and 800 Hz with a seismic moment between 10 and 10 Nm (corresponding to moment magnitudes of about $M_w = -0.73$ and 0.6, respectively). Most of the events within this cluster could be identified as daily production ripple-fired blasts at around 10 p.m. local time. The typical charge of an individual detonation is about 80 to 120 kg of explosives.

A second cluster (blue symbols) has corner frequencies between 100 and 400 Hz and a seismic moments between 10^{7} and 10^{9} Nm ($M_{\rm w}$ = -1.4 and 0, respectively). The associated events are mainly microseismic events and small rockbursts that occur at times randomly during the day. The source parameters were determined by the spectral fit method, but they are also consistent with the integration method, conducted previously.

A third small cluster (red symbols) has corner frequencies between 20 and 40 Hz and seismic moments between 10 8 and 10 9 Nm. These events also occur at times randomly through the day. About 90% of these events were recorded in January 2003, the first month of the network installation. Later on they can be found only sporadically in the data set. It is not yet clear if the high number of occurrence in January 2003 is related to the 26 January, 2003 M_w=1.1 rockbursts or, if the low number of occurrence later on is due to event-filter settings of the in-mine network processing routines. However, most of these low-frequency events are located within 100 meters of the large rockburst. Since the corner frequencies determined here are very low, the plateau part of the spectra is short and an accurate fit of the spectrum was not possible. Hence an underestimation of seismic moments and an overestimation of corner frequencies might be the case for this third cluster.

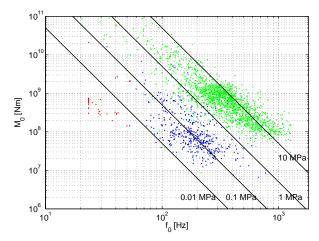


Figure 2. Relation between corner frequency and seismic moment for about 5,000 seismic events from the Pyhäsalmi mine. Blue indicates mining induced (stress release) events and green indicates delay-fired explosions. Red shows a small group of low-frequency events potentially associated with a large $(M_{\rm w}\,1.1)$ rockburst. Black lines indicate values of constant stress drop.

S/P ratios

Clear S waves are observed at the Pyhäsalmi in-mine network for both rockbursts and regular blasts. Figure 3 shows a typical seismic section for the first shot of a ripple-fired production blast, and we see that shear energy is clearly observed as close as 100 meters from the source.

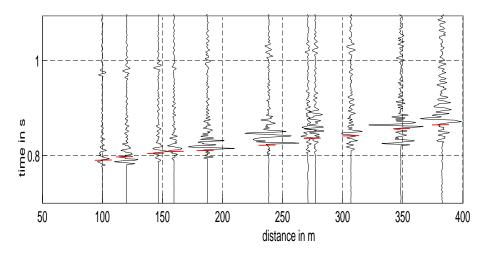


Figure 3. Observed seismograms from a production blast in the Pyhäsalmi mine showing how shear energy is observed as a function of distance from the source. The S arrival is indicated with red arrows. For the three-component sensors we show only the component having the largest S amplitude.

In order to assess the source contribution to the observed shear wave energy, we have calculated S/P ratios for both production blasts and rockbursts. A data set of about 2,000 well-located events with recordings at a minimum of 3 three-component stations was selected. The quality of the automatic processing results (i.e., phase picks, polarization estimates and event location) was checked by visual inspection of the three-component data, as shown in Figure 4. Events with erroneous time picks, polarization angles or location estimates were rejected.

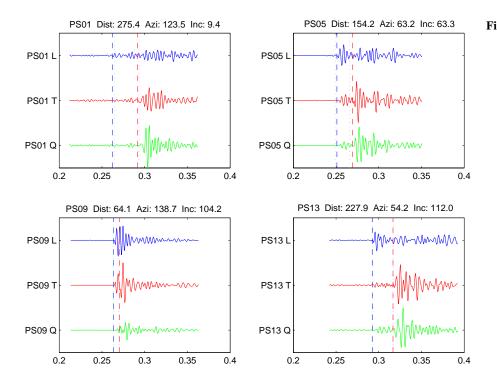


Figure 4. Panels showing observations and automatic processing results for a rockburst in the Pvhäsalmi mine. The data are filtered between 150 and 300 Hz. For each of the 4 three-component sensors we show the rayoriented L, T and Q components (blue, red, green). The rotation is based on the automatically estimated polarization angles of the P-phase, given above each panel. The automatic P- and S-picks are shown by blue and red dashed lines, respectively. The corresponding theoretical arrival times calculated from the automatic event location are shown by green and magenta dotted lines. The distance to the event (in meters) is given above each panel.

About 1,300 events remained after the quality control, and for these we measured the maximum P and S amplitudes in a series of one octave passbands. The list of passbands are given in Table 1

Table 1: Passbands (in Hz) used for measurement of P and S amplitudes

20-40	30-60	40-80	50-100	60-120
70-140	80-160	90-180	100-200	120-240
140-280	160-320	180-360	200-400	250-500
300-600	350-700			

The P amplitude was taken to be the maximum amplitude on the L-component. The S amplitude was taken to be the maximum of total S-wave amplitude in the orthogonal TQ plane. Due to the proximity of the sensors to the events, there is little time separation between the P and S-phases. For example, at a distance of 150 meters, the P and S-phases are separated by only 0.02 s, corresponding to a cycle with a frequency of 50 Hz. From inspection of the data filtered in the passbands given in Table 1, we have found that P and S do not separate sufficiently unless the lower cut-off of the passband is greater than approximately 1.5/(Sonset-Ponset), which in this case is 75 Hz. This implied that we could not obtain interpretable P and S amplitudes for the lowest passbands for recordings at the sensors closest to the events.

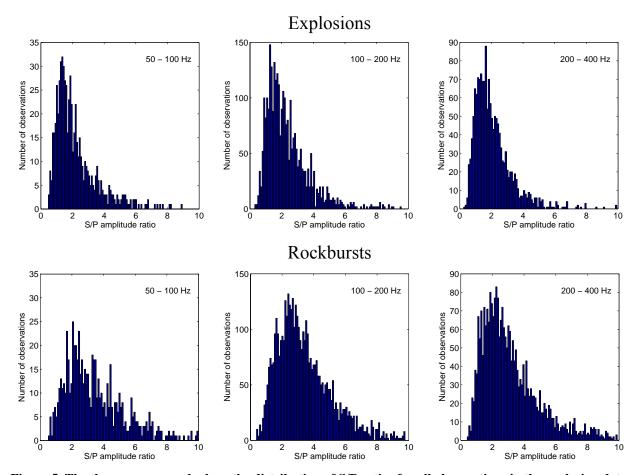


Figure 5. The three upper panels show the distribution of S/P ratios for all observations in the explosion data set for three frequency bands, 50-100 Hz, 100-200 Hz and 200-400 Hz. The three lower panels show the corresponding S/P ratios for the rockbursts.

The production blasting in the Pyhäsalmi mine takes place between 21:50 and 22:00 local time, so we labelled the events occurring within this time interval "the explosion data set". This data set probably also includes a few rockbursts, but these will not significantly influence the overall statistical characteristics of the explosions. In order to avoid effects like signals from rockfall and interference from previous detonations, we only considered the first detonation of ripple-fired blasts. Events occurring outside the time window 21:45 to 22:05 were denoted the "rockburst data set".

Figure 5 shows the distribution of S/P ratios for all observations of explosions and rockbursts for three frequency bands, 50-100 Hz, 100-200 Hz and 200-400 Hz. As expected, the explosions have generally lower S/P ratios for all frequency bands, but the difference is far too small to be significant for classification purposes. The maxima for the explosion distributions are all below 2, whereas they are all above 2 for the rockbursts. The rockbursts also have a wider distribution of S/P ratios, which can be explained by the variability of the radiation patterns from the rockburst sources. In addition, the rockbursts are more widely distributed within the mine volume, whereas the explosions only occur in the active parts of the mine.

The observed S/P ratios, for both explosions and rockbursts, do not show any significant dependency on the distance to the events. However, as discussed above, there is a lower distance limit below which the P and S-phases do not separate sufficiently in time to permit an estimation of the amplitudes. Example, for the 200-400 Hz passband this lower distance limit is about 50 meters. We have therefore no information in the distance dependency of the S/P ratios within the first tens of meters from the source.

In order to investigate S/P ratios for similar propagation paths within the mine, we have analyzed explosions and rockbursts occurring within a 100x100x100 meter volume, recorded at sensor PS01, at a distance of about 350 meters from the events. The distributions for this subset of the data set are very similar to those shown for the full data set in Figure 5. This indicates that the observed differences in S/P ratios between explosions and rockbursts are due to differences in the source characteristics, and not due to propagation effects along paths in the mine.

Wavefield modeling

We have investigated the physical mechanisms that partition seismic energy in the near source region by conducting modeling studies of the Pyhäsalmi mine. Our efforts have centered around two main activities: 1) comparison of observed data from seismic events at the Pyhäsalmi mine with synthetic data generated using a 3-D finite-difference wave propagation code and a 3-D geologic model of the mine; and 2) conducting synthetic sensitivity tests to determine the cause and characteristics of shear energy generated in the near source region.

The E3D finite-difference code and the 3-D geologic model of the mine have been described in more detail in previous papers (e.g., Larsen & Schultz, 1995; Bungum *et al.*, 2003). In summary, the wave propagation code solves the elasto-dynamic formulation of the wave equation on a staggered grid. As a full physics seismic simulation code, compressional, shear, and mode-converted energy is modeled. Viscoelastic attenuation capabilities are included. The code is computationally robust, and runs on a variety of high performance computing platforms. This makes it possible to simulate large problems. For this project, the E3D code utilizes a 3-D geologic model of the Pyhäsalmi mine. This model parameterizes the different geologic components of the mine (e.g., the ore bodies), as well as tunnels, voids, and back filled material produced by mine activity. The 3-D model is 500 m in all three dimensions. To date, the 3-D model has been discretized onto uniformly spaced grids of two and four meter grid spacings.

Figure 6 shows observed data at the Pyhäsalmi mine following a February 28, 2003 rockburst. These data are compared with synthetic data using the E3D code and the 3-D Pyhäsalmi geologic model discretized on the four meter grid. An explosive source was used for the simulation, with a frequency range up to 50 Hz. For comparison, the data have been low-pass filtered at 50 Hz. The rockburst event was located near the outer boundary of the mine ore body. The agreement between data and simulation is quite good. The synthetic data match the amplitude and overall character of the observed waveforms. There are a few discrepancies, which may be due to the simplicity of the source function, inaccuracies in the geologic model, and mislocated receiver (and source) positions.

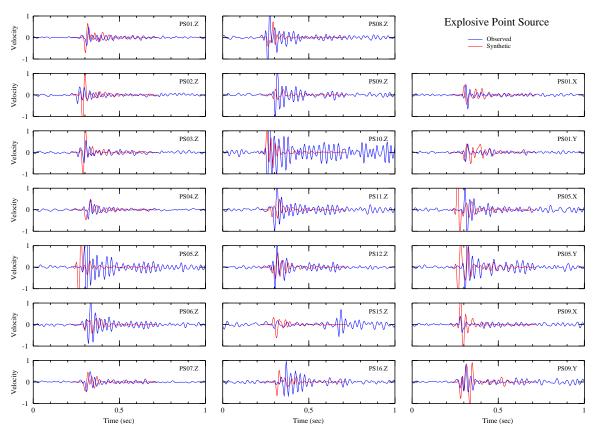


Figure 6. The blue seismograms show in-mine recordings in the Pyhäsalmi mine from a rockburst located near the outer boundary of the mine ore body. The data have been low-pass filtered at 50 Hz. Synthetic seismograms from an explosive source calculated in the Pyhäsalmi geologic model (4 m grid) using the E3D code are shown in red.

A similar comparison but one that incorporates seismic frequencies up to 100 Hz has also been made. In this case, the 2 meter discretized mine model was used. Although the agreement between model and observation was not quite as good as with the 50 Hz simulation, the comparison is still fair, especially considering the simplicity of the model. In addition to source and site effects, it may be more difficult to accurately characterize the Pyhäsalmi mine at these higher frequencies. Still, our results suggest that we can model real seismic events at the Pyhäsalmi mine, as well as perform synthetic sensitivity tests, at reasonably high levels of confidence.

To assess the sensitivity of the synthetic data to the source location, a simulation where the source position was modestly shifted from the rockburst location modeled above was performed. In this case, we moved the modeled source by 35 meters relative to its original location. The results of this simulation are shown in Figure 7. While a few stations indicate waveforms that are moderately different, in general, the agreement is very good. This suggests that a precise source location (i.e., less than 35 meters) is not required to model observed data from an event at the mine.

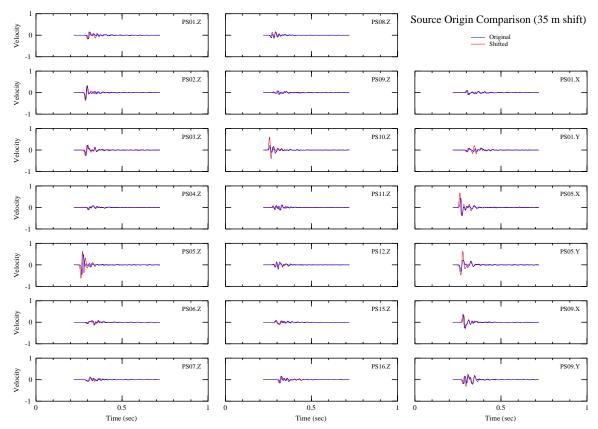


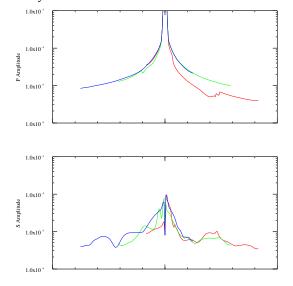
Figure 7. The seismograms shown in red are results from simulations after shifting the source location by 35 meters relative to its original location. The modeled seismograms using the original source location are shown in blue.

In addition to modeling the February 28, 2003 rockburst, several synthetic finite-difference simulations to investigate the character and cause of shear energy generation from the Pyhäsalmi mine were performed. In one series of simulations, we placed an explosive source at three locations in the mine region. One source was located at the boundary of the mine, in close proximity to the ore body. (In fact, this source was placed at the same location as that of the rockburst event discussed above). Another source was located near the center of the mine, while the third source was located relatively far from the mine boundary near the edge of our finite-difference numerical grid. The compressional nature of the explosive source (diagonal moment tensor components) means that all shear energy must be generated by mode converted P energy interacting with either the ore body or the structural components of the mine (e.g., tunnels, mined out regions). We investigated the amplitude and the near-source proximity of this shear energy generation. In each case, the geologic model discretized on the 4 meter grid was used.

Figure 8 shows the P and S potential amplitudes, and the P/S ratio, on the four horizontal and two vertical lines that extend outward from a source located at the rockburst location, near the boundary of the mine. The figure shows that shear energy is generated essentially in the immediate vicinity of the source. Both the P and S potentials fall off with distance due to geometrical spreading. The plot of P/S ratios indicates that most of the shear energy is generated within about 10 meters from the source, and that the shear energy is of comparable amplitude to the compressional energy.

For the source located at the center of the mine, we found that high amplitude shear energy is also generated in the immediate vicinity of the source. In this case, the modeled waveforms for both compressional and shear profiles are more complex than for a source at the rockburst location (in the outer part of the mine). This is likely due to the complexity of the mine (geologic heterogeneity, tunnels, voids).

For the source located some distance (over 100 meters) away from the mine, the modeling shows that little shear energy is generated. In fact, a good fraction of the shear energy that is observed may be numerical artifacts caused by unabsorbed reflections from the boundary of the finite-difference grid, as the source is relatively close to the grid boundary.



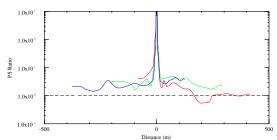


Figure 8. Modeled P amplitude (upper left panel), S amplitude (lower left panel) and P/S ratio (above) for an explosive source located at the rockburst location, near the boundary of the mine (see Figure 6). The calculations are made on the four horizontal and two vertical lines that extend outward from the source. The dashed line in the panel above indicates a P/S ratio of 1.

The Pyhäsalmi mine was also decomposed into its two main components to investigate the mechanism that generates shear energy. That is, we constructed one model that includes only the ore bodies, and another model that includes only the structural components of the mine (i.e., tunnels, mined regions). The second model is referred to as the "void" model. We ran simulations through both models using the 4 meter discretized grid. Close examination of the results from these simulations suggests that both the geologic inhomogeneity from the ore body and the structural voids may have similar influence on the mine-perturbed seismograms. It may be that the geologic inhomogeneity has a more pronounced effect early in the waveforms, whereas the voids are more influential and likely to produce scatter in the later portions of the seismograms. However, this may be due to the placement of the source near the ore body boundary, and further study is therefore needed to place more confidence in such a conclusion.

CONCLUSIONS AND RECOMMENDATIONS

We have continued our close cooperation with the operators of the Pyhäsalmi mine in central Finland. In late spring 2004, the in-mine network was extended by two triaxial sensors. Up to the end of April 2004, waveforms for more than 18,000 rockbursts and mining blasts were collected, and all of these events have been reprocessed using NOR-SAR's in-house microseismic monitoring software package (Oye & Roth, 2003). The automatic reprocessing has included phase onset picking, polarization analysis, phase association, event location, and estimation of seismic moment and corner frequencies for the located events. In addition, we have quality-checked the data and processing results for subsets of the 18,000 events, and created databases of high-quality events that have been used for both estimation of source parameters and measurements of S/P ratios.

The estimated seismic moments and corner frequencies for a database of high-quality events (about 4,000 explosions and about 1,000 rockbursts) show some scattering, but also three distinct clusters. The mining blasts have generally higher seismic moments than the rockbursts, and they also have generally higher corner frequencies. A third type of low-frequency rockburst is found with corner frequencies between 20 and 40 Hz and seismic moments similar to the production blasts. Most of these events occurred prior to, and in the same area as, a large rockburst (M_W 1.1) that occurred on 26 January 2003, which suggests that these events may be associated with a gradual failure of the pillar involved.

In order to assess the source contribution to the observed shear wave energy, we have calculated S/P ratios for both production blasts and rockbursts. A data set of about 2,000 well-located events with recordings at a minimum of 3 three-component stations has been selected. As expected, the rockbursts show generally larger S/P ratios for all fre-

quencies, but the difference is far too small to be significant for classification purposes. Significant shear energy is clearly observed within 50-100 meters of the source for both microearthquakes (rockbursts) and mine blasts. Due to the small time separation between the P- and S-phases, we were unable to measure S/P ratios for the very close distances (up to 30-50 meters). For distances above this range, no distance dependency could be observed for any type of events.

The modeling experiments have shown that we are able to model observed data at the Pyhäsalmi mine with a reasonable level of fit. The good match between synthetic and observed data lend a certain degree of confidence to our modeling techniques and our 3-D geologic model of the mine, which has been sampled with grid spacings of both 4 and 2 meters. Through simulations of the response to explosive sources, we have shown that significant shear energy can be generated in a 10-20 meter region around the source. This mode-converted energy can be of a similar amplitude to the incident compressional phase. This is consistent with the results obtained from analysis of events recorded at the inmine network, where we find distance independent S/P ratios for distances larger than 30-50 meters. Both the geologic heterogeneity and the structural influences of the mine appear to be contributors to the near-source generation of shear energy.

Based on updated information from the Pyhäsalmi mine operators, we plan to build a new model of the Pyhäsalmi mine, now with a grid spacing of 1 meter. With a 1 meter grid spacing we will have to include the smaller structures of the mine and its infrastructure, and the building of this model will thus require quite large efforts. With this model at hand, we plan to make more wavefield simulations using the E3D code, and again compare with shot data. We also plan to investigate the possibility of making a source moment tensor inversion of the events. The two new triaxial sensors of the in-mine network may be important for the success of such an effort.

A near-term objective is to characterize quantitatively the generation of shear energy due to seismic events located in and near the Pyhäsalmi mine. Specifically, we will perform a large number of E3D simulations to parameterize the generation of shear energy as a function of source distance, mine heterogeneity, and seismic frequency. Our preliminary results indicate that significant shear energy is created in a region that is well within a seismic wavelength from explosive sources located near mine heterogeneities. We will compare synthetically computed estimates of S/P ratio's with those observed from the mine data. This will serve both as a validation step and as a means to determine the fraction of shear energy that can be attributed to the source radiation pattern versus the fraction that can be attributed to mode converted energy at mine heterogeneities.

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